C.U.SHAH UNIVERSITY
Summer Examination-2017

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT1   **Branch: Diploma(All)**

Semester : 2   **Date : 04/05/2017**   **Time : 02:00 To 05:00**   **Marks : 70**

Instructions:
  (1) Use of Programmable calculator & any other electronic instrument is prohibited.
  (2) Instructions written on main answer book are strictly to be obeyed.
  (3) Draw neat diagrams and figures (if necessary) at right places.
  (4) Assume suitable data if needed.

Q-1   **Attempt the following questions:**   **(14)**
  a) If A(–7, 2) and B(3, 8) then midpoint of AB = _____.
      (a) $(-2,5)$ (b) $(5,-2)$ (c) $(2,5)$ (d) $(5,2)$
  b) x – intercept of line 2x – 6y + 4 = 0 is _____.
      (a) –2/3 (b) 2/3 (c) –2 (d) 2
  c) If A(– 3, 5) and B(2, – 4) are two points, find slope of AB = _____
      (a) –9/5 (b) 9/5 (c) – 5/9 (d) 5/9
  d) Centre of the circle $x^2 + y^2 = 25$ is _____ .

      (a) $(0,5)$ (b) $(5,0)$ (c) $(0,0)$ (d) None of these

  e) $\lim\limits_{x \to 0} x\left[\sqrt[x]{7} - 1\right] =$ _____

      (a) e (b) $\log_7 e$ (c) $\log_e 7$ (d) None of these

  f) $\lim\limits_{x \to 2} \dfrac{x^2 + x - 2}{x - 1} =$ _____

      (a) 1 (b) 0 (c) – 1 (d) None of these

  g) $\lim\limits_{x \to 0} \dfrac{\sin x}{x} =$ _____

      (a) – 1 (b) 0 (c) 1 (d) None of these

  h) $\dfrac{d(k)}{dx} =$ _____.   (Where k = constant)

      (a) 0 (b) kx^{k-1} (c) kx (d) None of these

  i) $\dfrac{d\left(\sin^2 x + \cos^2 x\right)}{dx} =$ _____

      (a) 2 (b) 1 (c) 0 (d) None of these

  j) $\dfrac{d(\tan x)}{dx} =$ _____

      (a) $-\cos ec^2 x$ (b) $\cos ec^2 x$ (c) $\sec^2 x$ (d) $-\sec^2 x$

k) If $f(x) = \log x$ then $f'(1) = $ _____

(a) 0 (b) $\dfrac{1}{2}\log 2$ (c) $2\log 2$ (d) 1

l) $\displaystyle\int \dfrac{1}{x^2+1}\,dx = $ _____

(a) $\tan^{-1} x + c$ (b) $\sin^{-1} x + c$ (c) $\cos^{-1} x + c$ (d) $\cot^{-1} x + c$

m) $\displaystyle\int \dfrac{1}{\sqrt{x^2-25}}\,dx = $ _____

(a) $\cot^{-1}\dfrac{x}{2} + c$ (b) $\tan^{-1}\dfrac{x}{2} + c$ (c) $\log\left|x+\sqrt{x^2-25}\right| + c$ (d) none of these

n) $\displaystyle\int_{2}^{5} \dfrac{1}{x}\,dx = $ _____

(a) $\log\dfrac{2}{5}$ (b) $\log\dfrac{5}{2}$ (c) $\log 10$ (d) None of these

Attempt any four questions from Q-2 to Q-8

Q-2	**Attempt all questions**	**(14)**
a)	Evaluate: $\displaystyle\lim_{\theta\to 0}\dfrac{\operatorname{cosec}\theta - \cot\theta}{\theta}$	**(5)**
b)	Prove that $\displaystyle\lim_{x\to 3}\dfrac{\sqrt{x+2}-\sqrt{5}}{\sqrt{x+4}-\sqrt{7}} = \dfrac{\sqrt{35}}{5}$	**(5)**
c)	If $(3, 8)$, $(4, 2)$ and $(-1, 5)$ are the vertices of a triangle, find the co ordinates of its centroid.	**(4)**
Q-3	**Attempt all questions**	**(14)**
a)	Find equation of a circle passing through points $(1, 0)$, $(0, 1)$ and $(0, 0)$.	**(5)**
b)	In which ratio Y – axis divides line segment joining points $(1, 2)$ and $(2, 1)$? Find co ordinates of division point.	**(5)**
c)	Evaluate: $\displaystyle\lim_{x\to(-1)}\dfrac{x^{41}+1}{x^{43}+1}$	**(4)**
Q-4	**Attempt all questions**	**(14)**
a)	Prove that the points $(0, -3)$, $(1, -2)$ and $(10, 7)$ are collinear.	**(5)**
b)	Find the equation of straight line passing through $(3, 4)$ and parallel to line $\dfrac{x}{2}+\dfrac{y}{2}=1$.	**(5)**
c)	If radius of a circle $x^2 + y^2 - 4x - 8y + k = 0$ is 4, find k.	**(4)**
Q-5	**Attempt all questions**	**(14)**
a)	Find derivative of $f(x) = \sqrt{x}$ using definition.	**(5)**
b)	Find $\dfrac{dy}{dx}$ if $y = \log\sqrt{\dfrac{1+\sin x}{1-\sin x}}$	**(5)**
c)	Prove that if $f(x) = \log\left(\dfrac{x-1}{x}\right)$ then prove that $f(x) + f(-x) = f(x^2)$.	**(4)**
Q-6	**Attempt all questions**	**(14)**

a) If $y = A\cos pt + B\sin pt$ then prove that $\dfrac{d^2y}{dt^2} + p^2y = 0$. **(5)**

b) Find $\dfrac{dy}{dx}$ if $e^x + e^y = e^{x+y}$. **(5)**

c) Evaluate: $\displaystyle\int \dfrac{1}{\sin^2 x \cos^2 x}\,dx$ **(4)**

Q-7 **Attempt all questions** **(14)**

a) Evaluate: $\displaystyle\int \dfrac{e^x(1+x)}{\cos^2(xe^x)}\,dx$ **(5)**

b) Evaluate: $\displaystyle\int x^n \log x\,dx$ **(5)**

c) Find $\dfrac{dy}{dx}$ if $y = \dfrac{x^2 - 1}{x^2 + 1}$. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Find the area of the standard circle $x^2 + y^2 = r^2$. **(5)**

b) Prove that $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\tan x}{\tan x + \cot x}\,dx = \dfrac{\pi}{4}$. **(5)**

c) $S = t^3 - 6t^2 + 9t + 6$ gives the distance travelled by a body in t seconds. Find velocity and acceleration at $t = 4$ seconds. **(4)**